Math 103 – Intermediate Algebra
Final Exam Review Problems Solutions

1. a) $\left(\frac{3a^{-2}b^3c}{6a^{-1}b^{-4}c^2}\right)^{-5} = \left(\frac{3ab^3b^4c}{6a^2c^2}\right)^{-5} = \left(\frac{b^7}{2ac}\right)^{-5} = \left(\frac{2ac}{b^7}\right)^5 = \frac{32a^5c^5}{b^{35}}$

   b) $\frac{10x^2y}{y^{-3}}\left(\frac{4x^{-2}}{15y^{-3}}\right) = \frac{10x^2yy^3}{1}\left(\frac{4y^3}{15x^2}\right) = \frac{10x^2y^4}{1}\left(\frac{4y^3}{15x^2}\right) = \frac{2y^4}{1}\frac{4y^3}{3} = \frac{8y^7}{3}$

   c) $\left(\frac{x^{7/3}}{x^{1/3}}\right)^{1/2} = \left(x^{7/3-1/3}\right)^{1/2} = \left(x^{6/3}\right)^{1/2} = \left(x^2\right)^{1/2} = x$

   d) $(-3a^{-1/3})^{-1}(9a^{2/3}) = \left(\frac{-3}{a^{1/3}}\right)^{-1}\left(9a^{2/3}\right) = \left(-\frac{a^{1/3}}{3}\right)\left(9a^{2/3}\right) = -\frac{9a^{1/3}a^{2/3}}{3} = -3a^{1/3+2/3} = -3a$

2. $-x^4 + 4x^3 + 3x^2 - 2x + 16$

3. a) $3x(3x^2 - 13x - 10) = 3x(3x + 2)(x - 5)$

   b) $(4x^2)^2 - (9y^2)^2 = (4x^2 - 9y^2)(4x^2 + 9y^2) = ((2x)^2 - (3y)^2)(4x^2 + 9y^2) = (2x - 3y)(2x + 3y)(4x^2 + 9y^2)$

   c) $(2z)^3 + 1^3 = (2z + 1)((2z)^2 - 2z\cdot 1 + 1^2) = (2z + 1)(4z^2 - 2z + 1)$

4. a) $(5x^4 - 6x^3 + x - 7) - (-2x^4 + 3x^3 + 4x^2 + 7x - 3) = 5x^4 - 6x^3 + x - 7 + 2x^4 - 3x^3 - 4x^2 - 7x + 3 =$

$$\begin{array}{r} 5x^4 - 6x^3 \qquad\quad + x - 7 \\ \underline{+\, 2x^4 - 3x^3 - 4x^2 - 7x + 3} \\ 7x^4 - 9x^3 - 4x^2 - 6x - 4 \end{array}$$

   b) $(x^2 + 3x - 2)(x - 5) = x^3 + 3x^2 - 2x - 5x^2 - 15x + 10 = x^3 - 2x^2 - 17x + 10$

   c) $(3x - 1)^2 = (3x - 1)(3x - 1) = 9x^2 - 6x + 1$

   d) $\frac{3}{x+3} - \frac{x+1}{(x+3)(x+2)} = \frac{3(x+2)}{(x+3)(x+2)} - \frac{x+1}{(x+3)(x+2)} = \frac{3x+6}{(x+3)(x+2)} - \frac{x+1}{(x+3)(x+2)} =$
   $\frac{3x+6-x-1}{(x+3)(x+2)} = \frac{2x+5}{(x+3)(x+2)}$

   e) $\frac{3(x+2)}{x(x+2)} - \frac{2x}{(x+2)x} = \frac{3x+6-2x}{(x+2)x} = \frac{x+6}{(x+2)x}$

   f) $\frac{(x+2)(x+2)}{(x-3)(x+2)} \cdot \frac{(2x+5)(x-3)}{x(x+2)} = \frac{(2x+5)}{x}$, $x \neq 3, -2$

g) $\dfrac{x^2-4}{12x+3}\cdot\dfrac{8x+2}{3x^2+5x-2}=\dfrac{(x+2)(x-2)}{3(4x+1)}\cdot\dfrac{2(4x+1)}{(3x-1)(x+2)}=\dfrac{2(x-2)}{3(3x-1)}$, $x\neq-\frac{1}{4},-2$

h) $4+3\sqrt{2}-2\sqrt{3}-5\sqrt{12}+\sqrt{18}+2\sqrt{25}=4+3\sqrt{2}-2\sqrt{3}-5\sqrt{4\cdot3}+\sqrt{9\cdot2}+2\sqrt{25}=$
$4+3\sqrt{2}-2\sqrt{3}-5\cdot2\sqrt{3}+3\sqrt{2}+2\cdot5=4+3\sqrt{2}-2\sqrt{3}-10\sqrt{3}+3\sqrt{2}+10=14+6\sqrt{2}-12\sqrt{3}$

i) $\left(7-\sqrt{3}\right)\left(\sqrt{6}+2\right)=7\sqrt{6}+14-\sqrt{18}-2\sqrt{3}=7\sqrt{6}+14-3\sqrt{2}-2\sqrt{3}$

5. a) $\dfrac{2x(x-2)}{(x-2)(x+1)}=\dfrac{2x}{x+1}$, $x\neq2$

b) $\dfrac{\frac{5}{x}-\frac{2x}{x}}{\frac{x\cdot x}{3x}-\frac{3\cdot3}{3x}}=\dfrac{\frac{5-2x}{x}}{\frac{x^2-9}{3x}}=\dfrac{5-2x}{x}\cdot\dfrac{3x}{x^2-9}=\dfrac{5-2x}{1}\cdot\dfrac{3}{x^2-9}=\dfrac{3(5-2x)}{x^2-9}$, $x\neq0$

c) $\dfrac{\frac{1}{x+2}}{\frac{4}{x^2-4}+\frac{x^2-4}{x^2-4}}=\dfrac{\frac{1}{x+2}}{\frac{4+x^2-4}{x^2-4}}=\dfrac{\frac{1}{x+2}}{\frac{x^2}{x^2-4}}=\dfrac{1}{x+2}\cdot\dfrac{x^2-4}{x^2}=\dfrac{1}{x+2}\cdot\dfrac{(x-2)(x+2)}{x^2}=\dfrac{x-2}{x^2}$, $x\neq-2, 2$

6. a) $\sqrt{(-7)^2}=\sqrt{49}=7$

b) $\sqrt{-7^2}=\sqrt{-49}=7i$

c) $\dfrac{\sqrt{49}}{\sqrt{121}}=\dfrac{7}{11}$

d) $\sqrt[3]{(-8)^3}=-8$

e) $-\sqrt[3]{-8}=-(-2)=2$

7. a) $y\cdot y^{1/2}=y^{1+1/2}=y^{3/2}$

b) $\dfrac{x^2}{x^{2/3}}=x^{2-2/3}=x^{6/3-2/3}=x^{4/3}$

c) $\left(a^{1/2}\cdot a^{3/4}\right)^2=\left(a^{1/2+3/4}\right)^2=\left(a^{2/4+3/4}\right)^2=\left(a^{5/4}\right)^2=a^{2\cdot(5/4)}=a^{5/2}$

8. a) $\sqrt{25\cdot2y^2x^4\cdot x}=5\,|\,y\,|\,x^2\sqrt{2x}$

b) $\sqrt[3]{27\cdot2a^6b^3\cdot b^2}=3a^2b\sqrt[3]{2b^2}$

c) $\dfrac{\sqrt[5]{y^5y}}{\sqrt[5]{32}}=\dfrac{y\sqrt[5]{y}}{2}$

9. a) $\dfrac{\sqrt{3}\sqrt{5x}}{\sqrt{5x}\sqrt{5x}}=\dfrac{\sqrt{15x}}{\left(\sqrt{5x}\right)^2}=\dfrac{\sqrt{15x}}{5x}$

b) $\dfrac{2\sqrt[3]{3}}{\sqrt[3]{3}\sqrt[3]{4x}}=\dfrac{2\sqrt[3]{2x^2}}{\sqrt[3]{2^2x}\sqrt[3]{2x^2}}=\dfrac{2\sqrt[3]{2x^2}}{\sqrt[3]{2^3x^3}}=\dfrac{2\sqrt[3]{2x^2}}{2x}=\dfrac{\sqrt[3]{2x^2}}{x}$

c) $\dfrac{1(3+\sqrt{7})}{(3-\sqrt{7})(3+\sqrt{7})} = \dfrac{(3+\sqrt{7})}{3^2+\sqrt{7}-\sqrt{7}-(\sqrt{7})^2} = \dfrac{3+\sqrt{7}}{9-7} = \dfrac{3+\sqrt{7}}{2}$

10. a) $\sqrt{-9\cdot 3} - \sqrt{-4\cdot 3} = 3\sqrt{3}i - 2\sqrt{3}i = \sqrt{3}i$

b) $5 - 5i + 3i - 3i^2 = 5 - 2i - 3(-1) = 5 - 2i + 3 = 8 - 2i$

11. a.) $\log_7 (7)^{-1/2} = -\frac{1}{2}$

b) $\log_2 2^6 = 6$

c) You can't have the logarithm of a negative number.

12. a) $\log_3 x^3 - \log_3 y^{1/2} - \log_3 z^2 = \log_3\left(\dfrac{x^3}{y^{1/2}z^2}\right) = \log_3\left(\dfrac{x^3}{z^2\sqrt{y}}\right)$

b) $\ln\left(\dfrac{x}{2z}\right) = \ln x - \ln 2z = \ln x - (\ln 2 + \ln z) = \ln x - \ln 2 - \ln z$

13. a) $5x - 35 = 3x + 2$

$2x = 37$

$x = \dfrac{37}{2}$

b) $4\left(\dfrac{x}{2} - 1\right) = 4\left(\dfrac{x}{4} + 2\right)$

$2x - 4 = x + 8$

$x = 12$

c) $|4x-3| + 2 = 7$

$|4x-3| = 5$

$4x - 3 = 5$ or $4x - 3 = -5$

$4x = 8$ or $4x = -2$

$x = 2$ or $x = -\frac{1}{2}$

d) $|2x-7| = -2$

no solution (The absolute value can't be negative.)

e) $x(2x - 9) = 5$

$2x^2 - 9x - 5 = 0$

$(2x + 1)(x - 5) = 0$

$2x + 1 = 0$ or $x - 5 = 0$

$x = -\frac{1}{2}, 5$

f) $2y - y^2 = 0$

$y(2 - y) = 0$

$y = 0$ or $2 - y = 0$

$y = 0, 2$

g) $x^3 - 5x^2 - x + 5 = 0$

$x^2(x - 5) - (x - 5) = 0$

$(x - 5)(x^2 - 1) = 0$

$(x - 5)(x - 1)(x + 1) = 0$

$x = 5, 1, -1$

h) $\left(\dfrac{2x}{x+3} + \dfrac{1}{x}\right)x(x+3) = 2x(x+3)$

$2x^2 + x + 3 = 2x^2 + 6x$

$3 = 5x$

$x = \frac{3}{5}$

i) $\frac{x}{x+1}+\frac{2}{x-5}=\frac{12}{(x-5)(x+1)}$

$\left(\frac{x}{x+1}+\frac{2}{x-5}\right)(x-5)(x+1)=\frac{12(x-5)(x+1)}{(x-5)(x+1)}$

$x(x-5)+2(x+1)=12$

$x^2-5x+2x+2=12$

$x^2-3x-10=0$

$(x-5)(x+2)=0$

$x=5,-2$

$x\neq 5$, so $x=-2$ is the solution.

j) $\left(\sqrt{x+7}\right)^2=(x-5)^2$

$x+7=x^2-10x+25$

$0=x^2-11x+18$

$0=(x-9)(x-2)$

$x=9, 2$

check: $\sqrt{9+7}=9-5$? $\sqrt{16}=4$, true

$\sqrt{2+7}=2-5$? $\sqrt{9}=-3$, not true

$x=9$ is the solution.

k) $\left(\sqrt{x+5}\right)^2=\left(\sqrt{x}+1\right)^2$

$x+5=x+2\sqrt{x}+1$

$4=2\sqrt{x}$

$2=\sqrt{x}$

$2^2=\left(\sqrt{x}\right)^2$

$x=4$

check: $\sqrt{4+5}=\sqrt{4}+1$? $\sqrt{9}=2+1$, true, so $x=4$.

l) Let $u=x^{1/5}$

$u^2-2u-15=0$

$(u-5)(u+3)=0$

$u=5,-3$

$x^{1/5}=5,\ x^{1/5}=-3$

$\left(x^{1/5}\right)^5=5^5,\ \left(x^{1/5}\right)^5=(-3)^5$

$x=3125,-243$

m) Let $u=(x^2-3)$

$u^2-7u+6=0$

$(u-6)(u-1)=0$

$u=6, 1$

$x^2-3=6,\ x^2-3=1$

$x^2=9,\ x^2=4$

$x=3,-3,2,-2$

n) $3^{x-5}=3^3$

$x-5=3$

$x=8$

o) $-7e^{-2x}+8=3$

$-7e^{-2x}=-5$

$e^{-2x}=\frac{5}{7}$

$-2x=\ln\frac{5}{7}$

$x=-\frac{\ln\frac{5}{7}}{2}\approx 0.1682$

p) $\log_5(x-3)=2$

$x-3=5^2$

$x-3=25$

$x=28$

q) $\log_2 x(x-2)=3$

$x(x-2)=2^3$

$x^2-2x-8=0$

$(x-4)(x+2)=0$

$x=4,-2$

$x\neq -2$, so $x=4$ is the solution.

14. a) $(x-3)^2=-\frac{7}{3}$

b) $2x+1=\pm\sqrt{9}$

$$x-3=\pm\sqrt{-\frac{7}{3}}$$

$$x=3\pm\sqrt{-\frac{7}{3}}=3\pm\sqrt{\frac{7}{3}}i$$

$$x=3\pm\frac{\sqrt{7}\sqrt{3}i}{\sqrt{3}\sqrt{3}}=3\pm\frac{\sqrt{21}}{3}i$$

$$2x+1=3,\quad 2x+1=-3$$
$$2x=2,\qquad 2x=-4$$
$$x=1,-2$$

15. a) (i) completing the square

$$x^2-x+\tfrac{1}{4}=1+\tfrac{1}{4}$$
$$\left(x-\tfrac{1}{2}\right)^2=\tfrac{5}{4}$$
$$x-\tfrac{1}{2}=\pm\frac{\sqrt{5}}{2}$$
$$x=\frac{1}{2}\pm\frac{\sqrt{5}}{2}=\frac{1\pm\sqrt{5}}{2}$$

(ii) quadratic formula:

$a = 1$, $b = -1$, $c = -1$

$$x=\frac{1\pm\sqrt{1-4(-1)}}{2}=\frac{1\pm\sqrt{5}}{2}$$

b) $2x^2 + 8x + 3 = 0$

(i) completing the square

$$x^2+4x+4=-\tfrac{3}{2}+4$$
$$(x+2)^2=\tfrac{5}{2}$$
$$x+2=\pm\sqrt{\frac{5}{2}}=\pm\frac{\sqrt{5}\cdot\sqrt{2}}{\sqrt{2}\cdot\sqrt{2}}=\pm\frac{\sqrt{10}}{2}$$
$$x=-2\pm\frac{\sqrt{10}}{2}=\frac{-4\pm\sqrt{10}}{2}$$

quadratic formula: $a = 2$, $b = 8$, $c = 3$

$$x=\frac{-8\pm\sqrt{64-4(2)(3)}}{2(2)}=\frac{-8\pm\sqrt{40}}{4}$$

$$x=\frac{-8\pm2\sqrt{10}}{4}=\frac{-4\pm\sqrt{10}}{2}$$

16. a) From the second equation $x = -4y$

Put this into the first equation: $2(-4y) - 3y = 11$

$-8y - 3y = 11 \Rightarrow -11y = 11 \Rightarrow y = -1$

$x = -4(-1) = 4$

$(4, -1)$

b) From the first equation $2y=3x-12$

$y=\tfrac{3}{2}x-6$

Put this into the second equation:

$4(\tfrac{3}{2}x-6)-6x=12$

$6x - 24 - 6x = 12 \Rightarrow -24 = 12 \Rightarrow$ no solution

17. a) Multiply the first equation by 10 to clear the decimals, multiply the second equation by -3 to cancel x and add.

$$\begin{array}{r} 3x+5y=14 \\ \underline{-3x-6y=-3} \\ -y=11 \end{array} \Rightarrow y=-11$$

Put this into one of the equations and solve for x.

$x + 2(-11) = 1$

$x - 22 = 1 \Rightarrow x = 23$

$(23, -11)$

b) Multiply the first equation by 4 and the second equation by -5 and add to cancel x.

$$\begin{array}{r} 20x+32y=56 \\ \underline{-20x+45y=85} \\ 77y=141 \end{array} \Rightarrow \tfrac{141}{77}$$

Put this into one of the equations and solve for x.

$5x+8\left(\tfrac{141}{77}\right)=14$

$5x=14-\tfrac{1128}{77}=-\tfrac{50}{77}\Rightarrow x=-\tfrac{10}{77}$

$\left(-\tfrac{10}{77},\tfrac{141}{77}\right)$

18. (1) Multiply the second equation by $-\tfrac{1}{3}$ and add it to the first to cancel x.

$x + 2y - z = -3$

$$\underline{-x \quad -y \qquad = -2}$$
$$y - z = -5$$

(2) Multiply the third equation by – 1 and add it to the first to cancel *x*.

$$x + 2y - z = -3$$
$$\underline{-x - 2y + 2z = 5}$$
$$z = 2$$

(3) Put this back into the equation obtained in step (1) to find *y*.

$$y - 2 = -5 \Rightarrow y = -3$$

(4) Put $y = -3$ and $z = 2$ back into any one of the original equations to find *x*.

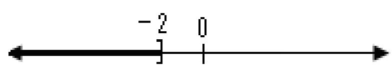
$$x + 2(-3) - 2 = -3 \Rightarrow x = 5$$

(5, – 3, 2)

19. a) $3 - 8x \geq 17 - x$

$-7x \geq 14$

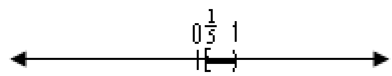
$x \leq -2$



b) $-5 < -5x \leq -1$

$1 > x \geq \frac{1}{5}$

$\frac{1}{5} \leq x < 1$



c) $|2x + 3| \leq 11$

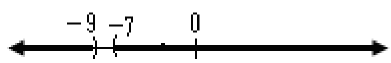
$-11 \leq 2x + 3 \leq 11$

$-14 \leq 2x \leq 8$

$-7 \leq x \leq 4$

d) $x + 8 < -1$ or $x + 8 > 1$

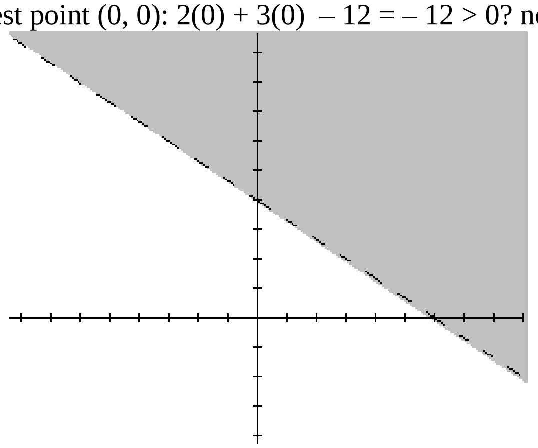
$x < -9$ or $x > -7$



20. a) *x*-intercept: $2x - 12 = 0 \Rightarrow x = 6$

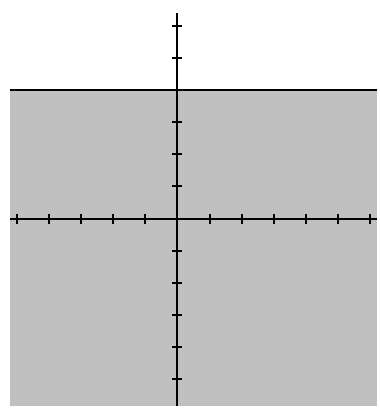
y-intercept: $3x - 12 = 0 \Rightarrow y = 4$

test point (0, 0): $2(0) + 3(0) - 12 = -12 > 0$? no.



b) $y = 4$ is a horizontal line 4 units up

The region satisfying $y \leq 4$ is the part under this line.



21. a) (i) $0 = 4 - x^2 \Rightarrow x^2 = 4 \Rightarrow x = \pm 2$

x-intercepts: (2, 0) and (– 2, 0)

$y = 4 - 0^2 = 4$

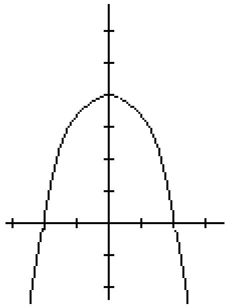
y-intercept: (0, 4)

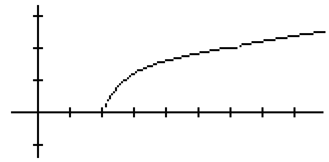
b) (i) $0 = \sqrt{x - 2}$

$0^2 = x - 2 \Rightarrow x = 2$

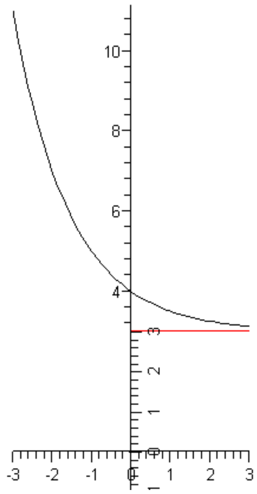
x-intercept: (2, 0)

$y = \sqrt{0 - 2} = \sqrt{-2} \Rightarrow$ no *y*-intercept

(ii) 

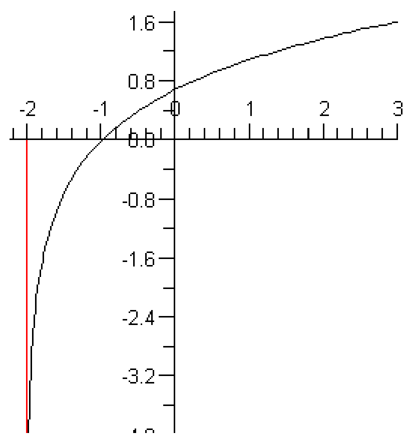
(ii) 

22. a) (i) $f(x) = 2^{-x} + 3$ is a shift of 3 units up from the function 2^x so the horizontal asymptote is $y = 3$
(ii) The negative sign in front of x means the graph is also reflected around the y-axis.



b) (i) $g(x) = \ln(x + 2)$ is a shift of 2 units left from the function $\ln x$ so the vertical asymptote is $x = -2$

(ii)



23. a) This is a function. Each first coordinate is paired with only one second coordinate. It doesn't matter that they are all paired with the same one.

b) This is not a function. 1 cannot be paired with both 5 and 7.

24. a) {2, 4, 6, 0} (the set of the first coordinates)

b) {3, 5, 7, 9} (the set of second coordinates)

25. a) i) $g(3) = -3^2 + 2(3) - 4 = -9 + 6 - 4 = -7$

ii) $g(2a) = -(2a)^2 + 2(2a) - 4 = -4a^2 + 4a - 4$
$g(1) = 1^2 + 2(1) - 4 = -1 + 2 - 4 = -3$
$g(2a) + g(1) = -4a^2 + 4a - 4 - 3 = -4a^2 + 4a - 7$

b) $$f(x) = \begin{cases} 2x - 1, & x < 1 \\ x + 5, & x \geq 1 \end{cases}$$

i) $f(0) = 2(0) - 1 = -1$

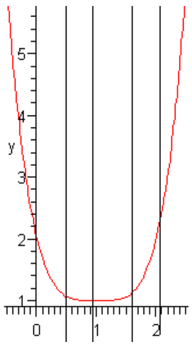
ii) $f(6) = 6 + 5 = 11$

ii) $f(-1) = 2(-1) - 1 = -3$
$f(1) = 1 + 5 = 6$
$f(-1) + f(1) = -3 + 6 = 3$

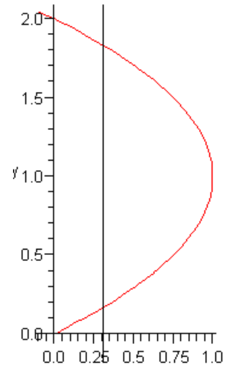
26. a) y is a function of x. No vertical line can

b) y is not a function of x. A vertical line can intersect

intersect the graph in more than one point.



the graph at two points.



27. $m = \dfrac{7-4}{1-(-5)} = \dfrac{3}{6} = \dfrac{1}{2}$

point-slope form: $y - 7 = \frac{1}{2}(x-1)$ or $y + 5 = \frac{1}{2}(x-4)$

Simplify either of these for general form: $2\big(y-7\big) = 2\big(\frac{1}{2}\big)(x-1)$

$2y - 14 = x - 1 \Rightarrow$ general form: $x - 2y + 13 = 0$

Solve for y for slope-intercept form: $2y = x + 13 \Rightarrow$ slope-intercept form: $y = \frac{1}{2}x + \frac{13}{2}$

28. Put $3x + 6y = 12$ in slope-intercept form to find its slope. $6y = -3x + 12 \Rightarrow y = -\frac{1}{2}x + 2 \Rightarrow$ the slope is $-\frac{1}{2}$

So the slope of a perpendicular line is 2.

Line through (3, 5) with slope 2: $y - 5 = 2(x - 3) \Rightarrow y - 5 = 2x - 6$

answer: $2x - y - 1 = 0$

29. $x - 2y = 5$ in slope-intercept-form is $y = \frac{1}{2}x - \frac{5}{2}$, so slope is $\frac{1}{2}$. So slope of a parallel line is $\frac{1}{2}$ also.

Line through (– 2, 4) with slope $\frac{1}{2}$: $y - 4 = \frac{1}{2}(x + 2) \Rightarrow y - 4 = \frac{1}{2}x + 1$

answer: $y = \frac{1}{2}x + 5$

30. $x = 4$

31. $y = 5$

32. $y = 3(x^2 - x + \frac{1}{4} - \frac{1}{4}) + 1 = 3(x - \frac{1}{2})^2 - \frac{3}{4} + 1 = 3(x - \frac{1}{2})^2 + \frac{1}{4}$, vertex: $\left(\frac{1}{2}, \frac{1}{4}\right)$

33. a) opens down because the leading coefficient is negative

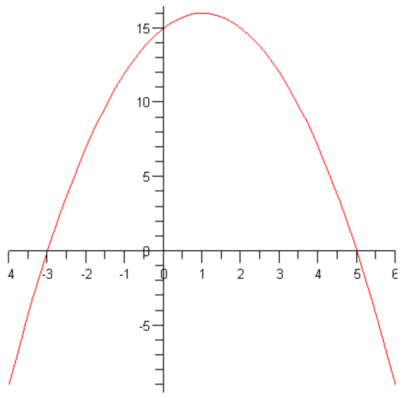
b) $f(0) = 15$, y-intercept: (0, 15)

c) $-x^2 + 2x + 15 = 0 \Rightarrow -(x^2 - 2x - 15) = 0 \Rightarrow -(x-5)(x+3) = 0 \Rightarrow x - 5 = 0$ or $x + 3 = 0$

x-intercepts: (– 3, 0) and (5, 0)

d) $x = \dfrac{-b}{2a} = \dfrac{-2}{-2} = 1$, $y = -1^2 + 2(1) + 15 = -1 + 2 + 15 = 16$, vertex: (1, 16)

e)



34. a) 1477.46 b) 50.75

35. Let $f(x) = 2x + 1$, $g(x) = x^2 - 5$. Find the following functions and simplify.

a) $(f \circ g)(x) = f(x^2 - 5) = 2(x^2 - 5) + 1 = 2x^2 - 10 + 1 = 2x^2 - 9$

b) $(g \circ f)(x) = g(2x + 1) = (2x + 1)^2 - 5 = 4x^2 + 4x + 1 - 5 = 4x^2 + 4x - 4$

c) $(f \circ f)(x) = f(2x + 1) = 2(2x + 1) + 1 = 4x + 2 + 1 = 4x + 3$

d) $x = 2y + 1 \Rightarrow x - 1 = 2y \Rightarrow \dfrac{x-1}{2} = y$

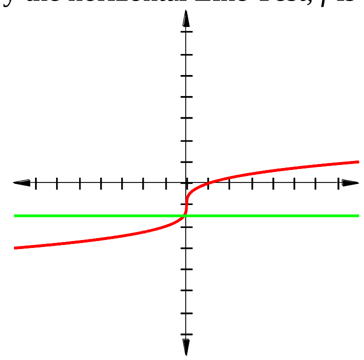
$$f^{-1}(x) = \frac{x-1}{2}$$

36.) $$(f \circ g)(x) = f\left(\frac{4x+2}{3}\right) = \frac{3\left(\frac{4x+2}{3}\right) - 2}{4} = \frac{4x+2-2}{4} = \frac{4x}{4} = x$$

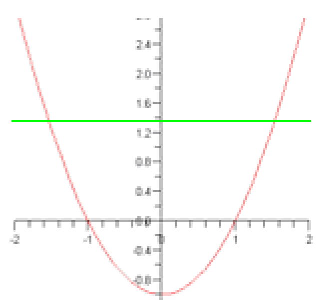
$$(g \circ f)(x) = g\left(\frac{3x-2}{4}\right) = \frac{4\left(\frac{3x-2}{4}\right) + 2}{3} = \frac{3x-2+2}{3} = \frac{3x}{3} = x$$

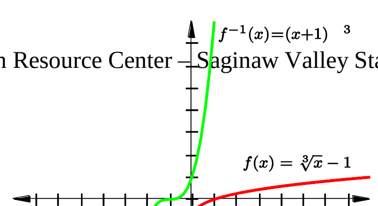
37. $\dfrac{\ln 7}{\ln 13} \approx .75865$

38. a) By the horizontal Line Test, f is one-to-one.



b) By the horizontal Line Test, f is **NOT** one-to-one.





$$y = \sqrt[3]{x} - 1$$
$$x = \sqrt[3]{y} - 1$$
$$\sqrt[3]{y} = x + 1$$
$$y = (x+1)^3$$
$$f^{-1}(x) = (x+1)^3$$

39. a) $A = \frac{bh}{2}$ First multiply both sides by 2:

$$2A = bh \Rightarrow$$
$$h = \frac{2A}{b}$$

b) $I = Prt$ Divide both sides by *Pr:*

$$t = \frac{I}{\mathrm{Pr}}$$

c) $P = 2l + 2w$

$$2l = P - 2w \Rightarrow$$
$$l = \frac{P - 2w}{2}$$

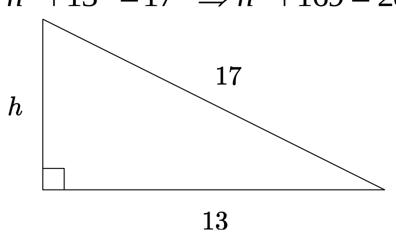
d) $S = C + Cr$

$$S = C(1+r) \Rightarrow$$
$$C = \frac{S}{1+r}$$

40. $F = \frac{k}{m^2}$. To find *k*, plug in *F* = 200 and *m* = 2: $200 = \frac{k}{2^2} \Rightarrow k = 800$.

Thus, $F = \frac{800}{m^2}$. When *m* = 5, $F = \frac{800}{25} = 32$.

41. $h^2 + 13^2 = 17^2 \Rightarrow h^2 + 169 = 289 \Rightarrow h^2 = 120 \Rightarrow h = \sqrt{120} \approx 11.0$ inches



42. Note that the highest point of this quadratic function is at its vertex. Thus, level of production that maximizes profit is given by the *x*-coordinate of the vertex: $h = \frac{-b}{2a} = \frac{-400}{2(-4)} = 50$ units.

The maximum profit is given by the *y*-coordinate of the vertex: $k = -3600 + 400(50) - 4(50)^2 = \$6400$.

43. The tip is $\$15-\$12.50=\$2.50$, so the rate is $\frac{2.50}{12.50}\cdot 100\% = 20\%$

44. Let *x* = the number of pounds of cheap coffee used by the grocer
Let *y* = the number of pounds of gourmet coffee used by the grocer.
Then $x+y=30$, since there are 30 lb. of house blend.
Also, the cost of the cheap coffee plus the cost of the gourmet coffee should give the cost of 30 lb of house blend: $3.5x+8y=4.25(30)$.
Now solve the equations simultaneously. (We used elimination.)

$$\left.\begin{array}{l} x+y=30 \\ 3.5x+8y=127.5 \end{array}\right\} \Rightarrow \left.\begin{array}{l} -8x-8y=-240 \\ 3.5x+8y=127.5 \end{array}\right\} \Rightarrow -4.5x=-112.5 \Rightarrow x=25$$

Since $x+y=30$, $y=30-x=30-25=5$.

Thus the grocer used 25 pounds of cheap coffee and 5 pounds of gourmet coffee.

45. This is a $distance = rate \times time$ problem. You can set up a chart to help you organize your thoughts.
Let *t* = the number of hours each car is traveling.

	Rate	Time	Distance
Car 1	40	t	$40t$
Car 2	60	t	$60t$

Thus $40t+60t=10 \Rightarrow 100t=10 \Rightarrow t=\frac{1}{10}$ hr = $\frac{1}{10}\cdot 60 = 6$ minutes. They are 10 miles apart after 6 minutes.

46. Let *x* be the number of hours it takes the two pigs working together to build a house.
Since the little pig builds 1 house in 4 hours, he builds $\frac{1}{4}$ of a house in 1 hour.

Since his brother builds1 house in 3 hours, the brother builds $\frac{1}{3}$ of a house in 1 hour.

Since together they build 1 house in *x* hours, together they build $\frac{1}{x}$ of a house in 1 hour.

Thus $\frac{1}{3}+\frac{1}{4}=\frac{1}{x}$. Multiplying each term by 12*x* gives: $4x+3x=12 \Rightarrow 7x=12 \Rightarrow x=\frac{12}{7}=1\frac{5}{7}$ hr.
(or approximately 1 hour and 43 minutes)

47. Let *h* be the height of the triangle, and let *b* be the length of its base. Then $b=h-3$

$A=\frac{1}{2}bh \rightarrow 20=\frac{1}{2}(h-3)(h) \rightarrow (h-3)(h)=40 \rightarrow h^2-3h-40=0$

$(h+5)(h-8)=0 \rightarrow$ Height must be positive therefore the height is 8 inches, and the base is 8 – 3 = 5 inches.

49. a) $4=-16t^2+8t+4 \rightarrow 0=-16t^2+8t \rightarrow 0=-8t(2t-1)\ t=0,\frac{1}{2}$

After ½ of a second, the height is 4 feet again.

b) $0=-16t^2+8t+4 \rightarrow 0=-4(4t^2-2t-1)$

Since $4t^2-2t-1$ doesn't factor, we need to use the quadratic formula with $a=4, b=-2, c=-1$:

$t=\frac{2\pm\sqrt{2^2-4(4)(-1)}}{2\cdot 4}=\frac{2\pm\sqrt{20}}{8}=\frac{2\pm 2\sqrt{5}}{8}=\frac{1\pm\sqrt{5}}{4}\approx 0.809,-0.309$. Since t must be positive,

$t=\frac{1+\sqrt{5}}{4}\approx .809\,\text{sec}$

50. a) $5=10e^{k\cdot 70} \Rightarrow 0.5=e^{70k} \Rightarrow \ln(0.5)=\ln(e^{70k}) \Rightarrow \ln(0.5)=70k \Rightarrow k=\frac{\ln(0.5)}{70}\approx -.0099$

b) Plugging in t = 100 and the *k* that we found in part a), we get: $A=10e^{\frac{\ln(0.5)}{70}\cdot 100}\approx 3.71$ grams.

51. P is the initial amount. Thus if you double the amount of money in the bank, you will end up with $2P$. So:

$2P=Pe^{0.025t}$. Dividing both sides by P gives: $2=e^{0.025t} \Rightarrow \ln 2=\ln e^{0.025t} \Rightarrow \ln 2=0.025t$

Thus $t=\frac{\ln 2}{0.025}\approx 27.73$ years.